Unit 7 Mathematical Calculations For Science

Assignment booklet (P1, P2, P3, P4, P5)

Indices P1

Starter questions

A

What is:

1. 2^2=
2. 2^3=
3. 2^4=
4. 2^5=
5. 5^3=

E

Investigate what happens when you work out:

1. $2^2 \times 2^2$=
2. $2^2 \times 2^3$=
3. $2^5 \div 2^4$=
4. $2^2 \div 2^2$=

Discuss what you notice

C

Copy and highlight:

1. $A^b \div A^c = A^{b-c}$
2. $A^b \times A^c = A^{b+c}$
3. $(A^b)^c = A^{bc}$
4. $A^0 = 1$
5. A^{-b}=

Main questions

1. **Simplify:**
   a. $A^2 \times A$
   b. $A^4 \times A^2$
   c. $A^8 \times A^2$
   d. $3A^9 \times A^3$
   e. $4A^2 \times 5A^1$
2. **Simplify:**
   a. $A^{10} \div A$
   b. $A^{21} \div A^7$
   c. $A^{20} \div A^4$
   d. $9A^{12} \div 3A^6$
   e. $12A^6 \div 4A^4$

4. **Simplify:**
   a. A^0
   b. B^0
   c. 99^0
   d. 52^0
   e. 0.061^0

1. **Simplify:**
   a. A^{-4}
   b. A^{-6}
   c. A^{-2}
   d. $2A^{-4}$
   e. $9A^{-3}$

Paste

Star questions:

Use a scientific calculator to try to find out what a power of ½ (or 0.5) does, (try try numbers 2, 4, 9, 16), now try the power of one third (try the numbers 8, 27, 64).

Now try to work out and write down what these powers do:

Logarithms P1

Become Friends With Logarithms

Logarithms are actually very easy once you get used to how they behave. After that they are as predictable as yesterday's weather. All it takes is practice.

What is a logarithm? A logarithm is an exponent. Specifically, logarithms are written "$\log_b x = y$" and read as "the log, base *b*, of *x* is *y*". It *really* means "base *b*, taken to the power *y*, gives *x*". Hence,

$$\log_b x = y \Leftrightarrow b^y = x$$

So, for instance, if you had an unknown variable as an exponent, you'd work out a logarithm to find that exponent's value. Here's an easy example: $2^x = 8$. In this case we know from past experience that $x = 3$, since $2^3 = 8$. Hence, "3 is the logarithm, base 2, of 8", or $3 = \log_2 8$.

- ***Practice Set I – Converting from exponential into logarithm form.***

In this set, you take a known expression that should be obviously true, and convert it into its equivalent logarithm form. This will give you practice to see where the proper places are for each number in the logarithm form.

Example: Given $4^2 = 16$, you'd rewrite this as $\log_4 16 = 2$.

$2^4 = 16$	$5^3 = 125$
$3^2 = 9$	$7^0 = 1$
$10^3 = 1000$	$8^{1/3} = 2$
$25^{1/2} = 5$	$2^{-1} = \frac{1}{2}$
$10^{-2} = 0.01$	$3^{-3} = \frac{1}{27}$

- ***Practice Set II – Converting from logarithm form back into exponential form.***

Now we go backwards, so to speak. Convert these true logarithms back into their equivalent exponential format. At first you may not see that each given logarithm is true, but if you rewrite it as the more familiar exponential form, hopefully this will help you see how logs are properly written.

Example: $\log_6 36 = 2$ is a true logarithm statement. Trust me. If you rewrite this as $6^2 = 36$, perhaps you will agree now, too.

$\log_2 32 = 5$	$\log_9 81 = 2$	$\log_2 \frac{1}{4} = -2$
$\log_5 25 = 2$	$\log_{16} 4 = \frac{1}{2}$	$\log_{49} 7 = \frac{1}{2}$
$\log_{10} 10 = 1$	$\log_6 1 = 0$	$\log_8 2 = \frac{1}{3}$

- ***Practice Set III – Find the error!***

In this set, you have logarithms that seem to have all the right numbers but they are in the wrong places. You must re-arrange them so that the logarithm is corrected.

Example: $\log_2 3 = 8$ is incorrect. Literally translated into exponentials, this would read as $2^8 = 3$, which is obviously false (2 to the 8th power is 256, which is not equal to 3, of course). But, you know that $2^3 = 8$, so the corrected logarithm should be $\log_2 8 = 3$

$\log_5 2 = 25$	$\log_{25} \frac{1}{2} = 5$	$\log_7 1 = 7$
$\log_4 0 = 1$	$\log_{10} 2 = 100$	$\log_4(-1) = \frac{1}{4}$
$\log_{64} 8 = 2$	$\log_3 5 = 125$	$\log_3 3 = 27$

- ***Practice Set IV – Find the Unknown.***

In each logarithm is an unknown value *x*. Determine its value. You may have to rewrite back into exponential form, but with practice you should be able to determine *x* without having to do this middle step.

Example: $\log_7 49 = x$. Rewrite as $7^x = 49$, which implies $x = 2$.

$\log_x 25 = 2$	$\log_4 2 = x$	$\log_{12} x = 1$
$\log_6 x = 2$	$\log_3 x = -1$	$\log_{10} 0.001 =$
$\log_8 64 = x$	$\log_9 1 = x$	$\log_x \frac{1}{125} = -3$

Note: In many books you will often see expressions like $\log_4 16$. In this case simply set it equal to a variable *x* and perform the steps as in the above set. You now have $\log_4 16 = x$.

Note: Square roots are equivalent to the $\frac{1}{2}$ power, cube roots are equivalent to the $\frac{1}{3}$ power, and so forth. Also recall that negative powers act as reciprocals, so that $3^{-1} = \frac{1}{3}$, $3^{-2} = \frac{1}{9}$, et cetera. Many people stumble on these because they may have forgotten the basic rules of exponents. For example, $\log_5 \sqrt{5} = \frac{1}{2}$ and $\log_6 \frac{1}{36} = -2$. These are both true. Do you see why? If not, re-arrange into exponential form and see if that helps.

Logarithms and exponentials "undo" one another, just as squares and square roots undo one another. In algebra, if we need to "undo" an exponential, we apply a logarithm. Equivalently, if we need to "undo" a logarithm, we apply an exponential (usually we just re-write as an exponential). This leads to a handful of useful properties: (Remember, base *b* is always positive and $\neq 1$)

$\log_b b^x = x$. In this case, the "$\log_b$" cancels the base *b*, and leaves *x* all by itself.
$b^{\log_b x} = x$. This is a restatement of the above property. "Bases cancel logs".

Properties, continued.

$\log_b b = 1$, for all bases *b*. Rewrite into exponential form and this will be obvious. It is a corollary of the first property.
$\log_b 1 = 0$. This is true because $b^0 = 1$ for all bases *b*.

Algebraic properties of logarithms. There are three, and you must know these. This is how you actually manipulate logarithms as an algebraic operation.

$I.\quad \log_b(xy) = \log_b x + \log_b y$

$II.\quad \log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$

$III.\quad \log_b x^n = n\log_b x$

All three properties have to do with how exponents behave when you multiply or divide terms with a common base.

Lastly, we note a couple of conventions:

"log" without a base written is understood to be $\log_{10}$.
"ln" is shorthand for the natural logarithm, which is the $\log_e$.

- ***Practice Set V – Expansions of Logarithms.***

Example: Expand $\log_2\left(\frac{4x^2y}{\sqrt{z}}\right)$. By rule II, we have $\log_2 4x^2y - \log_2 z^{1/2}$ (Do you see where the ½ power of *z* came from?). By Rule I we can break up the first term and get

$$\log_2 4x^2y - \log_2 z^{1/2} = \log_2 4 + \log_2 x^2 + \log_2 y - \log_2 z^{1/2}.$$

Now we can apply Rule III, and simplify the first term (remember, $\log_2 4 = 2$) to get

$$2 + 2\log_2 x + \log_2 y - \tfrac{1}{2}\log_2 z$$

Try these:

$\log_3(9xy^2)$

$\log_4(xy^2\sqrt{z})$

$\log_5\left(\frac{x}{25y}\right)$

$\log\left(\frac{100x^2}{y^3}\right)$

$\log_6(6x^2y)^3$

$\log\left(\frac{10(x-1)^2}{yz^2}\right)$

- ***Practice Set VIII – (Calculus) Differentiation of Logarithm Functions***

The basic derivative form is $\frac{d}{dx}[\ln u] = \frac{1}{u} \cdot \frac{du}{dx}$, where u is some function of x. The expression u is called the argument.

Example: Find the derivative of $y = \ln(x^2 + 3x - 5)$. Using the above form, we have $y' = \frac{1}{x^2+3x-5} \cdot (2x + 3)$, or more simply as $y' = \frac{2x+3}{x^2+3x-5}$.

Use logarithm properties (Practice Set V) to "break apart" a complicated argument. The individual smaller logarithms are then easier to differentiate.

Example: Find the derivative of $y = \ln\left(\frac{x^2(3x-1)}{x^5+7}\right)$. Expand this logarithm: we have $\ln\left(\frac{x^2(3x-1)}{x^5+7}\right) = 2\ln x + \ln(3x - 1) - \ln(x^5 + 7)$. Therefore, the derivative is found by differentiating each of these smaller logarithms: $y' = \frac{2}{x} + \frac{3}{3x-1} - \frac{5x^4}{x^5+7}$.

Find the derivatives:

$y = \ln(5x + 2)$
$y = \ln(x^4 + 2x^2 + 5)$
$y = \ln(e^{2x} + 7)$
$y = \ln\left(x^2\sqrt{3x + 1}\right)$
$y = \ln\left(\frac{x^4}{2x^3-5}\right)$
$y = \ln\left(\frac{7x^2\sqrt{5x+1}}{e^{2x}(x^4+1)}\right)$

Algebra P2

Solve (find x)

1. $2x + 5 = 33$
2. $6x - 6 = 12$
3. $10x + 5 = 75$
4. $12x - 9 = 51$
5. $20 + 4x = 36$

Work out:

1. **$3 - 8$**
2. **$9 - 13$**
3. **$-4 + 7$**
4. **$-3 - 9$**

List as many solutions to the equation $2x + y = 20$ as you can, for example

X=5 and y = 10 because 2x 5 + 10 = 20

Can you think of 10?

Can you include negative numbers?

<u>Main questions</u>

1. **Find a and b for each pair of simultaneous equations:**
   a. **$5a + 2b = 14$ $\quad$ $6a + 2b = 16$**
   b. **$7a + 3b = 27$ $\quad$ $6a + 3b = 24$**
   c. **$10a - 2b = 30$ $\quad$ $3a - 2b = 2$**
   d. **$9a - 6b = 42$ $\quad$ $6a - 6b = 18$**
2. **Find a and b for each pair of simultaneous equations:**
   e. **$4a + 7b = 27$ $\quad$ $4a - 7b = 13$**
   f. **$3a + 2b = 35$ $\quad$ $2a - 2b = 10$**
   g. **$11a - 8b = 4$ $\quad$ $a + 8b = 44$**
   h. **$5a + 3b = 69$ $\quad$ $7a - 3b = 75$**
3. **Find a and b for each pair of simultaneous equations:**
   i. **$5a + 6b = 28$ $\quad$ $6a + 2b = 18$**

<u>Star question</u>

In a shop there are no prices up, but if Tom is charged £1.80 for 3 cakes and 2 cups of Tea and Andy buys 4 cakes and 4 cups of tea for £4, how much does 1 cup of tea and 1 cake cost?

Solving Quadratic Equations

1) $(x + 2)(x - 3) = 0$	2) $(x + 6)(x - 2) = 0$	3) $(5x + 20)(x + 1) = 0$
4) $x^2 + 5x + 6 = 0$	5) $x^2 - 5x + 6 = 0$	6) $x^2 + 5x + 4 = 0$
7) $x^2 - 7x + 10 = 0$	8) $x^2 + 9x + 20 = 0$	9) $x^2 + 7x + 12 = 0$
10) $x^2 - 3x - 10 = 0$	11) $x^2 + 5x - 6 = 0$	12) $x^2 - 9x - 10 = 0$
13) $x^2 + 5x - 14 = 0$	14) $x^2 - 8x - 20 = 0$	15) $x^2 - 4x - 21 = 0$
16) $x^2 - 6x + 9 = 0$	17) $x^2 - 10x + 25 = 0$	18) $x^2 - 4x - 32 = 0$

Circular Measure P3

Trigonometry P4

Convert each degree measure into radians and each radian measure into degrees.

1) $325°$

2) $340°$

3) $60°$

4) $-\frac{4\pi}{3}$

5) $\frac{23\pi}{12}$

6) $\frac{10\pi}{3}$

7) $570°$

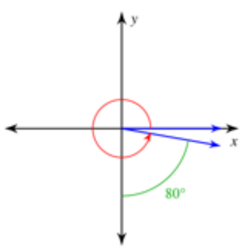
8) $-315°$

9) $\frac{\pi}{2}$

10) $-180°$

Find the measure of each angle.

11)



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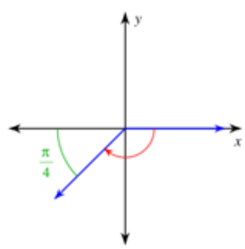

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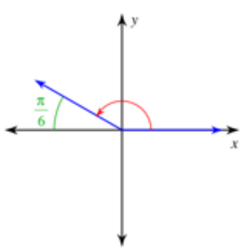

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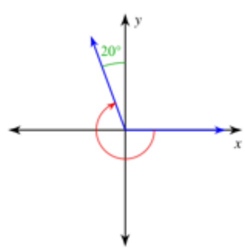

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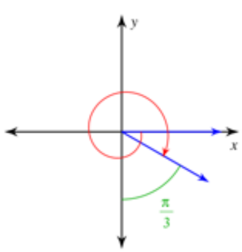

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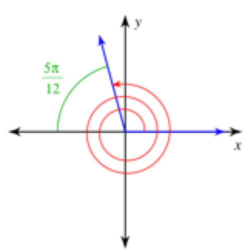

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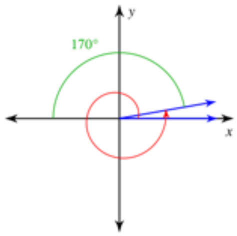

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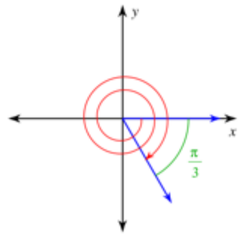
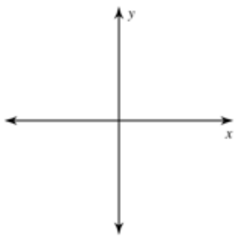
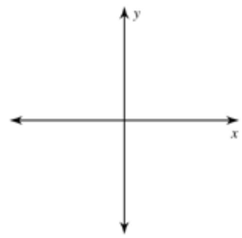


Draw an angle with the given measure in standard position.

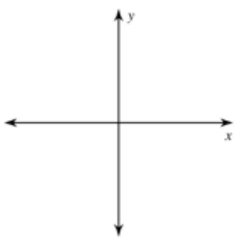
19) $280°$



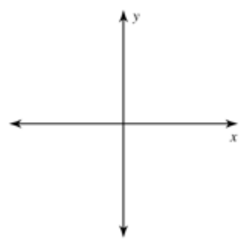
20) $710°$



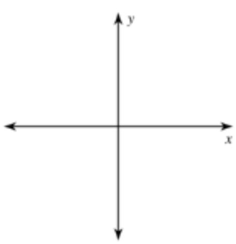
21) $-120°$



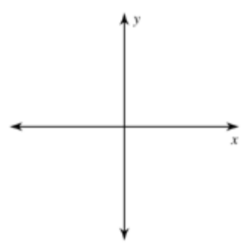
22) $\frac{11\pi}{6}$



23) $-\frac{10\pi}{3}$



24) $440°$



State the quadrant in which the terminal side of each angle lies.

25) $-509°$

26) $-\frac{5\pi}{6}$

Find the missing side. Round to the nearest tenth.

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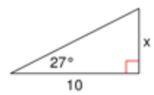

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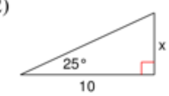

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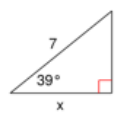

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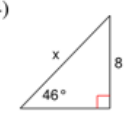

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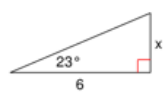

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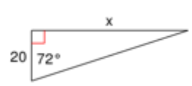

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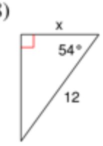

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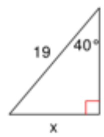

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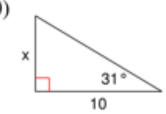

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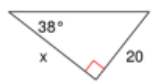

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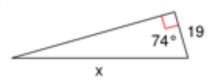

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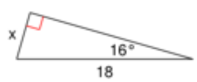

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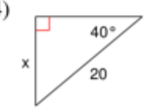

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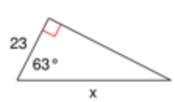

18)

AP Calculus Worksheet on Growth and Decay

Solve each:

1. A certain city had a population of 25,000 in 1960 and a population of 30,000 in 1970. Assume that its' population will continue to grow exponentially at a constant rate. What population can its' city planners expect in the year 2010?

2. In a certain culture of bacteria, the number of bacteria increased sixfold in 10 hours. How long did it take to double their number?

3. Carbon extracted from an ancient skull contained only 1/6 as much radioactive C^{14} as carbon extracted from a present-day bone. How old is the skull? The half-life is 5730 years.

4. Radioactive radium has a half-life of 1620 years. What percent of a given amount remains after 100 years?

5. Suppose you discover in your attic an overdue library book on which your great-great-grandfather owed a fine of 30 cents 100 years ago. If an overdue fine grows exponentially at a 5% annual rate compounded continuously, how much would you have to pay if you returned the book today?